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# Distributed Generation Investment by a Microgrid Under Uncertainty

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**ABSTRACT.** This paper examines a California-based microgrid's decision to invest in a distributed generation (DG) unit that operates on natural gas. While the long-term natural gas generation cost is stochastic, we initially assume that the microgrid may purchase electricity at a fixed retail rate from its utility. Using the real options approach, we find natural gas generating cost thresholds that trigger DG investment. Furthermore, the consideration of operational flexibility by the microgrid accelerates DG investment, while the option to disconnect entirely from the utility is not attractive. By allowing the electricity price to be stochastic, we next determine an investment threshold boundary and find that high electricity price volatility relative to that of natural gas generating cost delays investment while simultaneously increasing the value of the investment. We conclude by using this result to find the implicit option value of the DG unit.

**KEYWORDS.** Distributed Generation; Real Options; Optimal Investment.

## 1. INTRODUCTION

The objective of deregulated electricity sectors is to improve economic efficiency by providing market signals to participants (Wilson, 2002). Participants in competitive electricity markets should make decisions better suited to their circumstances than ones in the regulated paradigm. Traditionally, the viewpoint was that the electricity sector exhibits characteristics of a “natural monopoly,” i.e., costs that decline with output. This necessitated a vertically integrated structure with franchise monopolies operating in distinct regions. Specifically, the investment and operating complementarities between generation and transmission of electricity were internalised under the auspices of vertically integrated investor-owned utilities (IOUs) subject to government regulation. Such an industrial structure, however, turned the potentially competitive generation function of the sector into a *de facto* monopoly. Indeed, little evidence exists that large generation companies are necessary to achieve economies of scale (Joskow, 1987). Due to inefficiencies of the structure, such as low capital and labour productivity along with a gap between retail and wholesale prices (Deng, 1999), many jurisdictions have deregulated their electricity sectors over the past twenty years. In a broad sense, such measures have kept functions with “natural monopoly” characteristics, such as distribution and transmission, under the control of regulatory agencies, while opening up the generation and retailing functions to competition. As with other sectors of the economy, greater efficiency may then be achieved by matching demand and supply in a decentralised fashion such that consumers and producers make decisions based on their own utility and profit-maximising objectives.

The deregulation experiment also includes scope for decentralisation of the traditional power grid by involving microgrids, which are localised networks of distributed generation (DG) and combined heat and power (CHP) applications matched to local energy requirements. Facilitated by such dispersion, a greater proportion of electricity generation takes place closer to loads than in the familiar centralised regulated paradigm (Pepermans et al., 2005). Microgrids utilising small-scale, on-site DG offer not only tangible benefits stemming from the possibility of lower-cost electricity and greater system energy efficiency via CHP applications, but also “qualitative” improvements, such as power quality and reliability (PQR) more customised to enduse requirements and improved prospects for the adoption of small-scale renewable energy technologies, which do not fit easily into large-scale centralized organisations. PQR benefits are characterised as *qualitative* here not because they are without genuine economic value but rather because methods for incorporating their contribution to microgrid value streams is currently rudimentary at best. Due to deregulation, market participants must also manage risk from uncertain fuel prices that could have been passed on in the era of vertically integrated utilities. Furthermore, considerable regulatory barriers still exist that inhibit more widespread adoption of DG, ranging from poorly defined and enforced interconnection standards to tariff components such as back-up charges and exit fees. Considerable research and folklore exists on the struggles that self-generators face when trying to coexist in the power system with enormous entrenched electricity utilities (Alderfer et al 2000). While the analysis conducted here is purely economic, it should be noted that the diffusion of emerging microgrid technologies will likely be determined as much by the regulatory environment as by economic fundamentals, as is true with many aspects of electricity markets.

From a microgrid’s perspective, making investment and operational decisions concerning DG units should involve an assessment of the uncertainty in both electricity and fuel prices. While the former are largely fixed in the case of utility-provided time-of-use (TOU) tariffs, the latter may be subject to considerable volatility, being typically subject to monthly procurement cost revision. A microgrid should account for this uncertainty when making investment and operational decisions. Whereas in previous studies, we have strived to model the economics and thermodynamics of a microgrid in a detailed, but purely deterministic setting (Siddiqui et al., 2005a and Siddiqui et al., 2005b), here, we incorporate randomness into our analysis necessitating abstraction from some real-world considerations. Specifically, in this paper, we examine the problem of a California-based microgrid via the real options approach to determine fuel cost thresholds below which it is optimal for the microgrid to install DG. We next extend the basic model to allow for operational flexibility and also evaluate the option to disconnect entirely from the utility. Finally, we allow the electricity price to be stochastic in order to examine its impact on the microgrid’s investment decision. Where possible, we contrast the results yielded by the real options approach with that implied by a traditional, deterministic discounted cash flow (DCF) analysis.

## 2. PROBLEM FORMULATION

We assume that a microgrid has a constant load,  $\frac{Q}{8760}$  (kW), that it must serve via either utility purchases or DG output over which it has the discretion to invest. The turnkey cost of the DG unit,  $I$  (US\$), is deterministic and includes all capacity and installation costs. For now, we assume a deterministic utility electricity price,  $P$  (US\$/kWh), whereas the long-term natural gas generation cost,  $C$  (US\$/kWh)<sup>1</sup>, evolves according to a geometric Brownian motion (GBM) process as follows (Näsäkkälä and Fleten, 2005):

$$dC = \alpha C dt + \sigma C dz \quad (1)$$

This implies that successive percentage changes in  $C$  are independent of each other, which is a reasonable model for the long-term evolution of commodity prices (Schwartz and Smith, 2000). Here,  $\alpha$  is the annual growth rate of  $C$ ,  $\sigma$  is the annual volatility of  $C$ , and  $dz$  is the increment to a standard Wiener process. We additionally define the annual real risk-free interest rate as  $r$  and let  $\delta$  be the convenience yield associated with natural gas generation, i.e., the risk-adjusted rate of return on natural gas generation minus its growth rate. Intuitively, this is the opportunity cost of retaining the option to invest. For a financial call option,  $\delta$  is the forgone dividend. Finally, we assume that once the DG unit is installed, its effective lifetime is infinite due to the possibility of maintenance upgrades. This simplification is further justified by the fact that the discrepancy between the present value (PV) of a perpetuity and the PV of an annuity due decreases with the length of the time horizon. For example, if the actual lifetime of a DG unit is thirty years, then the ratio of the latter to the former per dollar for  $\delta = 0.04$  is

$$\left( \frac{1}{\delta} - \frac{1}{\delta(1+\delta)^{30}} \right) \delta(1+\delta) = 0.7192.$$

Given price uncertainty and managerial flexibility, we use the real options approach (Dixit and Pindyck, 1994) to model the microgrid's decision-making. This approach is appropriate because it trades off in continuous time the PV of benefits from investment with its associated costs. Specifically, the real options approach includes not only the tangible investment costs, but also the opportunity costs of exercising the option to invest. Indeed, at times, it may be better to retain the option to invest even for an apparently profitable project from the DCF perspective. Following from the pricing of financial call options (Black and Scholes, 1973), the real options approach constructs a risk-free portfolio using a short position on the underlying asset and then equating its expected appreciation (net of any dividend payments) to the instantaneous risk-free rate that could have been earned by investing in the portfolio. For a perpetual option, the resulting partial differential equation (PDE) becomes an ordinary differential equation (ODE), which is solved analytically using boundary conditions. As part of the solution, an investment threshold price for the underlying asset is obtained, above which investment is triggered. If an investment opportunity also offers embedded options, such as the discretion to abandon or to suspend and re-start production, then the approach is still valid, but may not yield closed-form analytical solutions. Nevertheless, the threshold prices and resulting option values may be obtained numerically. In Section 3.3, we also consider a case with stochastic electricity prices, but we first turn to the base case with the electricity price fixed.

### 3. NUMERICAL EXAMPLES

#### 3.1. Case 1: Option to Invest

We first consider a simple case in which the microgrid has the option to invest in a 500 kW DG unit without any operational flexibility. Specifically, assume that there are two states of the world: one in which the microgrid holds the option to invest in DG and the other in which it has exercised the option. In the former situation, the microgrid receives no incremental electricity cost savings, and in the latter, its PV of cost savings is  $\frac{P}{r} - \frac{C}{\delta}$  per kWh. Note that the natural gas cost is discounted using the risk-adjusted convenience yield to account for the uncertainty in  $C$ . Letting  $V_0(C)$  and  $V_1(C) = \frac{P}{r} - \frac{C}{\delta}$  be the PVs per kWh of the microgrid's cost savings without and with DG installation, respectively, we now construct a risk-free portfolio,  $\Phi$ , consisting of one unit of  $V_0(C)$  and short  $V_0'(C)$  units of natural gas (kWh equivalent). By

equating the expected appreciation of  $\Phi$  less any dividend payments to the instantaneous risk-free return on an investment of amount  $\Phi$ , we obtain:

$$r\Phi dt = E[d\Phi] - \delta V_0'(C)dt \quad (2)$$

In order to simplify the right-hand side of Equation (2), we first note that:

$$\begin{aligned} \Phi &= V_0(C) - CV_0'(C) \\ \Rightarrow d\Phi &= dV_0 - V_0'(C)dC \end{aligned} \quad (3)$$

Next, we apply Itô's Lemma to obtain:

$$dV_0 = V_0'(C)dC + \frac{1}{2}V_0''(C)(dC)^2 \quad (4)$$

By substituting Equation (4) into Equation (3) and taking expectations, we obtain:

$$\begin{aligned} d\Phi &= \frac{1}{2}V_0''(C)(dC)^2 \\ \Rightarrow E[d\Phi] &= E\left[\frac{1}{2}V_0''(C)(dC)^2\right] \\ \Rightarrow E[d\Phi] &= \frac{1}{2}V_0''(C)\sigma^2 C^2 dt \end{aligned} \quad (5)$$

Finally, we substitute Equation (5) into to the right-hand side of Equation (2) and re-arrange to obtain the following ODE:

$$\frac{1}{2}\sigma^2 C^2 V_0''(C) + (r - \delta)CV_0'(C) - rV_0(C) = 0 \quad (6)$$

Since  $\lim_{C \rightarrow \infty} V_0(C) = 0$ , the solution to Equation (6) is:

$$V_0(C) = A_2 C^{\beta_2} \quad (7)$$

Here,  $A_2 > 0$  and  $\beta_2 = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} - \sqrt{\left(\frac{(r - \delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$  are constants, with  $A_2$  to be determined endogenously<sup>2</sup>. Note that Equation (7) implies that the value of the option to invest is high when the natural gas generating cost is low. Using the following value-matching and smooth-pasting conditions, we solve for  $A_2$  and the investment threshold cost,  $C_I$ :

$$V_0(C_I) = V_1(C_I) - \frac{I}{Q} \quad (8)$$

$$V_0'(C_I) = V_1'(C_I) \quad (9)$$

Equation (8) states that upon exercise, the microgrid receives a cash flow equal to the PV from an installed DG unit minus the investment cost. As for Equation (9), it is a technical condition that requires the two value curves to meet tangentially at the point of exercise; indeed, if they do not, then the holder of the option would be better off exercising it either earlier or later.

The parameters for this and subsequent cases (unless otherwise indicated) are given in Table 1 and correspond roughly to the situation for a microgrid in the service territory of San Diego Gas and Electric (SDG&E) utility during the year 2000 (Siddiqui et al., 2005b). Using these data, we find that  $\beta_2 = -4.2405$ , while the closed-form solutions to Equations (8) and (9) reveal that

$C_I = \frac{\beta_2 \delta}{\beta_2 - 1} \left( \frac{P}{r} - \frac{I}{Q} \right) = 0.0772$  and  $A_2 = -\frac{1}{\delta \beta_2 C_I^{\beta_2 - 1}} = 8.7453 \times 10^{-6}$ . In other words, the microgrid should install a 500 kW DG unit only if the natural gas generation cost decreases to US\$0.0772/kWh. As illustrated in Fig. 1, the value to retain the option to invest in DG is greater than the net present value (NPV) of the cost savings from installed DG as long as the natural gas cost is greater than US\$0.0772/kWh. At  $C_I$ , the value of the option is exactly equal to the NPV of cost savings, thereby triggering investment. Note that  $V_0(C)$  is defined only over  $C \geq C_I$  since the option to invest is exercised for any generation cost below the threshold value.

*Table 1. Base Case Parameter Values.*

Parameter	Value
$P$	US\$0.10/kWh
$I$	US\$0.50M
$\frac{Q}{8760}$	500 kW
$\sigma$	0.06
$\delta$	0.04
$r$	0.04

For comparison, a completely deterministic evaluation of this investment opportunity recommends waiting until the natural gas cost drops to  $C_I^{\text{det}} = P - \frac{rI}{Q} = 0.0954$  before installing DG. This is because the deterministic DCF decision rule is to invest as long as:

$$\begin{aligned}
 \frac{P}{r} - \frac{C}{r} - \frac{I}{Q} &\geq 0 \\
 \Rightarrow \frac{C}{r} &\leq \frac{P}{r} - \frac{I}{Q} \\
 \Rightarrow C_I^{\text{det}} &= P - \frac{rI}{Q}
 \end{aligned} \tag{10}$$

Intuitively, Equation (10) states that investment occurs if the cost of on-site generation plus the amortised investment cost per kWh of the DG unit is less than the electricity price. Since this approach neglects the opportunity cost of waiting before investing, it suggests investing sooner than the real options approach. Consequently, its PV per kWh from cost savings is also smaller:

$$\frac{P}{r} - \frac{C_I^{\text{det}}}{r} = 0.1142 \text{ compared to } \frac{P}{r} - \frac{C_I}{\delta} = 0.57.$$

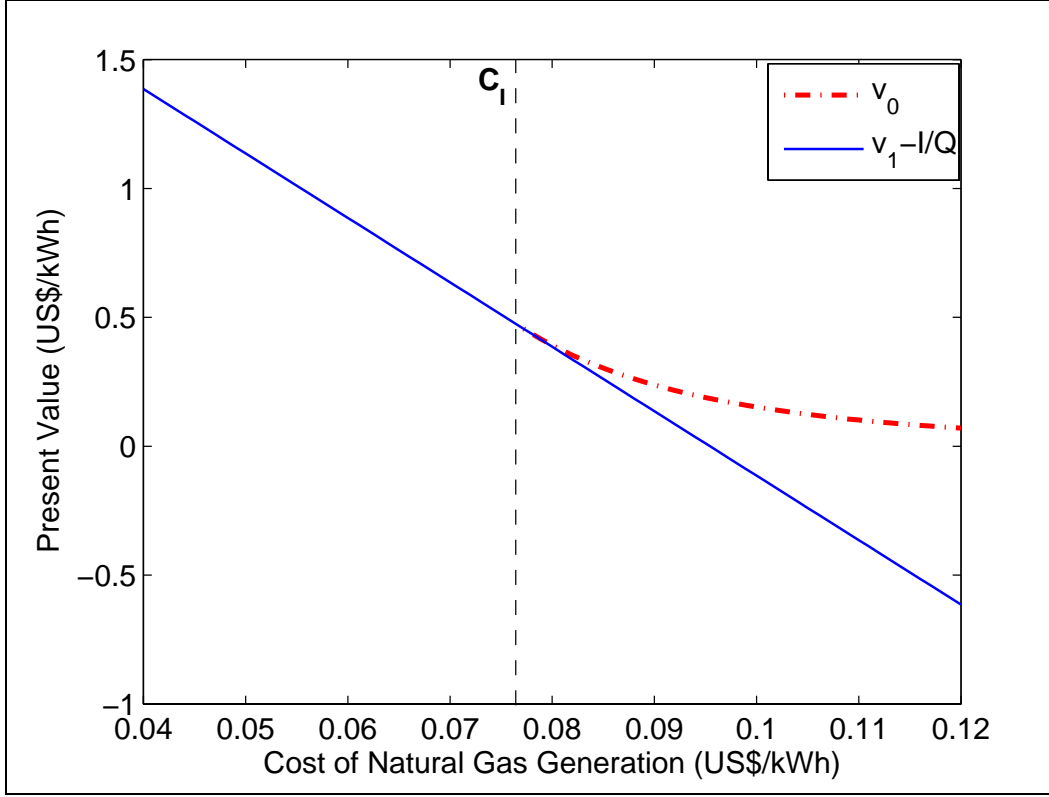


Figure 1. Value of Investment Opportunity.

### 3.2. Case 2: Operational Flexibility and the Option to Disconnect

Now suppose that the microgrid has the options to shutdown and re-start the installed DG unit at costs  $S$  and  $R$  (both in US\$), respectively, as well as the option to disconnect permanently from the utility. We, thus, have four states in this system as opposed to two in Section 3.1:

1. DG is not installed
2. DG is installed and operational with utility connection available
3. DG is installed and switched off with utility connection available
4. DG is installed and operational without utility connection

In the first state, the PV of the microgrid's cost savings is simply  $V_0(C) = A_2' C^{\beta_2}$ . In the second state, the microgrid may exercise the option to suspend DG operation if the cost of natural gas generation becomes sufficiently high relative to utility-provided electricity. Alternatively, if the cost of natural gas generation decreases sufficiently relative to  $P$ , then the microgrid may exercise the option to disconnect permanently from the utility. Otherwise, if it operates the DG unit, then it obtains the PV of cost savings relative to utility electricity purchases. Therefore, the value of the microgrid's cost savings in the second state is:

$$V_1(C) = B_1 C^{\beta_1} + B_2 C^{\beta_2} + \frac{P}{r} - \frac{C}{\delta} \quad (11)$$

Here,  $B_1 > 0$ ,  $B_2 > 0$ , and  $\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} = 5.2405 > 1$  are constants, with  $B_1$  and  $B_2$  endogenous to the system. Note that the first term of Equation (11) is the option



to shut down, the second term is the option to disconnect, and the remaining terms comprise the PV of cost savings per kWh from using DG rather than utility purchases.

In a suspended state, the microgrid's PV of cost savings reflects the option to re-start:

$$V_2(C) = D_2 C^{\beta_2} \quad (12)$$

Again,  $D_2 > 0$  is an endogenous constant, and Equation (12) indicates that when DG operation is suspended, the microgrid does not accrue any cost savings and re-starts the DG unit when the cost of natural gas generation decreases sufficiently. Finally, from the operational state, the microgrid may also choose to disconnect permanently from the utility should the cost of natural gas generation drop significantly. If it exercises this option, then the PV of its costs savings is:

$$V_3(C) = \frac{P}{r} - \frac{C}{\delta} + \frac{X}{rQ} \quad (13)$$

Here  $X$  (in US\$) is the annual customer charge paid to the utility. For SDG&E, this was approximately US\$50 per month, which implies  $X = 600$ . Given the value functions in Equations (7), (11), (12), and (13), we also require the following value-matching and smooth-pasting conditions (as well as the ones in Equations (8) and (9)) describing transitions among the states<sup>3</sup>:

$$V_1(C_S) = V_2(C_S) - \frac{S}{Q} \quad (14)$$

$$V_1'(C_S) = V_2'(C_S) \quad (15)$$

$$V_2(C_R) = V_1(C_R) - \frac{R}{Q} \quad (16)$$

$$V_2'(C_R) = V_1'(C_R) \quad (17)$$

$$V_1(C_X) = V_3(C_X) \quad (18)$$

$$V_1'(C_X) = V_3'(C_X) \quad (19)$$

These may be solved numerically for  $A'_2$ ,  $B_1$ ,  $B_2$ ,  $D_2$ ,  $C'_I$ , the shutdown and re-start cost thresholds,  $C_S$  and  $C_R$ , respectively, and the disconnect cost threshold,  $C_X$ .

Since the resulting system of equations is highly non-linear, there is no analytical solution to it. Nevertheless, for the parameters in Table 1 and  $S = 5000$ ,  $R = 5000$ , and  $X = 600$ , we numerically obtain  $A'_2 = 1.0721 \times 10^{-5}$ ,  $B_1 = 45085$ ,  $B_2 = 1.712 \times 10^{-9}$ ,  $D_2 = 1.4896 \times 10^{-5}$ ,  $C'_I = 0.085$ ,  $C_S = 0.104$ ,  $C_R = 0.096$ , and  $C_X = 0.038$ <sup>4</sup>. These results imply that operational flexibility makes it easier for the microgrid to invest by increasing the NPV of an installed DG unit since the microgrid could always revert to utility purchases in case of sustained increases in the natural gas generation cost. In addition, the microgrid waits until the natural gas generation cost is well above (below)  $P$  before turning off (on) the DG unit. This reflects not only the explicit cost  $S$  ( $R$ ) of turning off (on) an active (inactive) DG unit, but also the implicit cost  $R$  ( $S$ ) that the microgrid incurs in the future if it ever re-starts (shuts down) an inactive (active) DG unit. Although the option to disconnect is available, it does not seem likely to be exercised in the short term because the PV of its benefit, i.e., the cost savings from not having to pay the utility's customer charge, is outweighed by the option to buy electricity from the utility. Therefore, the disconnection cost threshold is set much lower than  $P$ ; effectively, the microgrid

must expect sustained positive cash flows from DG operation in the future before exercising the disconnection option.

Fig. 2 illustrates the value of the re-start option along with  $V_0(C)$  and  $V_1(C)$ . Again, we draw the curves only over the regions in which they are defined. Specifically,  $V_0(C)$ ,  $V_1(C)$ ,  $V_2(C)$ , and  $V_3(C)$  exist over  $C \geq C'_I$ ,  $C_S \geq C \geq C_X$ ,  $C \geq C_R$ , and  $C \geq C_X$ , respectively. Since  $C_X$  is very low, neither it nor  $V_3(C)$  is indicated on the graph. We observe from Equations (11) and (13) that  $V_1(C)$  is almost linear, i.e., similar to  $V_3(C)$ , for low  $C$ , and then resembles the option to re-start,  $V_2(C)$ , for intermediate  $C$ . Furthermore, at each threshold price, the discontinuous jump between the appropriate curves is equal to the fixed cost per kWh of switching states. For example, at  $C'_I$ , it is optimal to invest in the DG unit, i.e., jump from curve  $V_0(C)$  to  $V_1(C)$ , at which point the difference between the two curves reflects the investment cost per kWh.

As indicated in Section 3.1, greater uncertainty also makes the microgrid more hesitant to act, whether to invest, suspend, re-start, or disconnect. Fig. 3 illustrates the effect of natural gas generation cost volatility on the relevant thresholds. In particular, greater volatility at a relatively high natural gas cost not only increases the option value to invest, but also delays investment as the microgrid waits for the natural gas cost to fall sufficiently in order to ensure that any cost decrease is sustainable. On the other hand, if the natural gas cost is relatively low, then greater volatility makes an installed DG unit less valuable as the microgrid becomes more likely to face extremely high and unbounded costs while simultaneously experiencing extremely low, but bounded (by zero), costs. Since the effect of the former is stronger than that of the latter, the microgrid is more reluctant to invest. Therefore, higher natural gas cost volatility makes investment in DG both more valuable and less likely<sup>5</sup>. This also has the effect of delaying any decision to suspend DG operation because the implicit reward of shutting down, i.e., the option to re-start DG, increases with natural gas volatility. Similarly, disconnecting completely from the utility becomes highly improbable for even a moderate level of volatility. Hence, there is a wider zone of inaction as volatility increases.

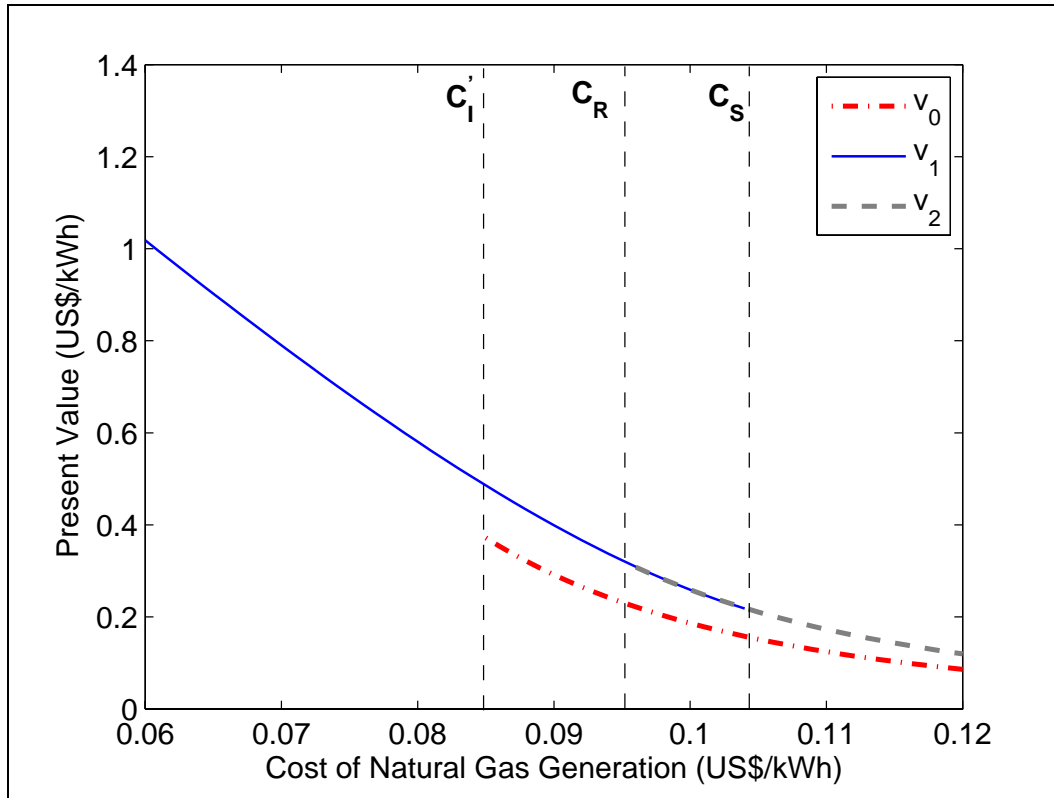


Figure 2. Value of Investment Opportunity (Including Options to Shutdown, Re-start, and Disconnect).

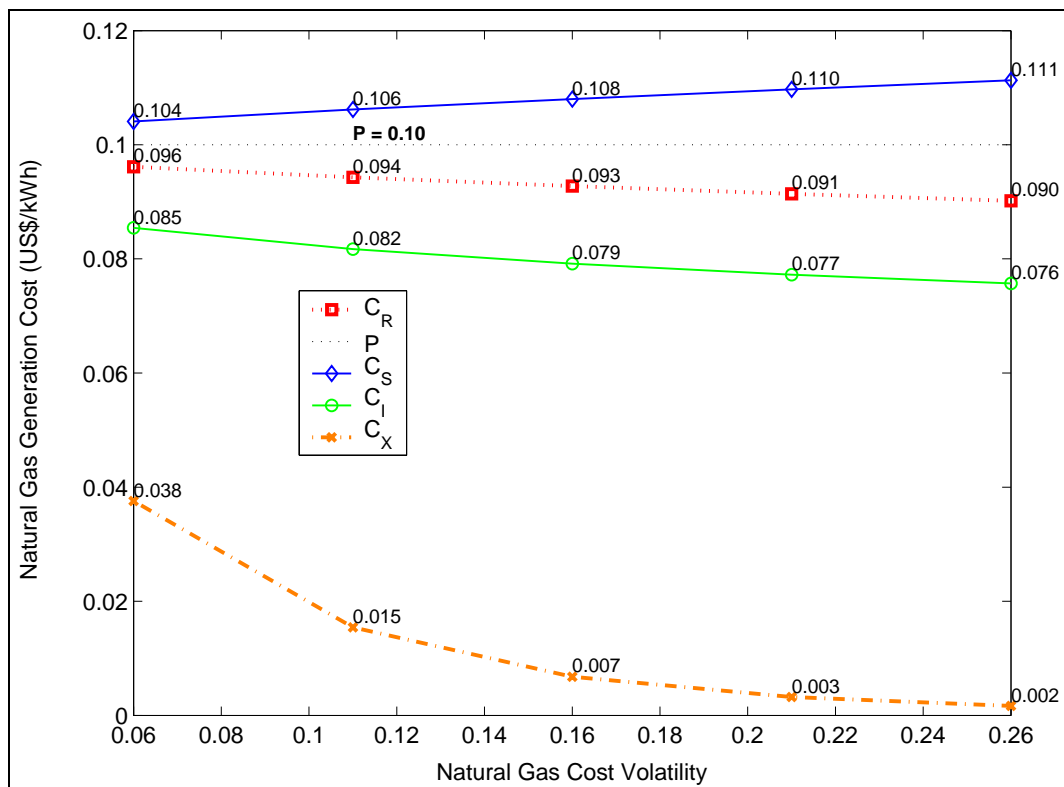


Figure 3. Investment, Shutdown, Re-start, and Disconnection Cost Thresholds.

### 3.3. Case 3: Stochastic Electricity Price

We now allow the electricity price to evolve according to a GBM process as follows:

$$dP = \alpha_p P dt + \sigma_p P dz_p \quad (20)$$

In addition, we assume that the electricity price and the cost of natural gas generation have instantaneous correlation  $\rho dt$ . Proceeding analogously to Section 3.1, we construct a risk-free portfolio by using the option to invest in DG,  $F(P, C)$ , along with short positions on both  $P$  and  $C$ . By equating the instantaneous risk-free rate of return on the portfolio with the expected appreciation in the portfolio less any dividend payments, we can write a PDE for the investment problem of the microgrid (assuming no operational flexibility exists). Letting  $p \equiv \frac{P}{C}$ ,  $F(P, C) = Cf(p)$ , and assuming the option value is homogeneous in  $(P, C)$ , we obtain the following ODE (Dixit and Pindyck, 1994):

$$\frac{1}{2}(\sigma_p^2 + 2\rho\sigma_p\sigma + \sigma^2)p^2 f''(p) + (\delta - \delta_p)pf'(p) - \delta f(p) = 0 \quad (21)$$

The closed-form solution to this after applying the value-matching and smooth-pasting conditions  $f(p_I) = \frac{p_I}{\delta_p} - \frac{1}{\delta}$  and  $f'(p_I) = \frac{1}{\delta_p}$ , respectively, is  $f(p) = a_1 p^{\gamma_1}$ , where

$$p_I = \frac{\gamma_1 \delta_p}{(\gamma_1 - 1)\delta} \text{ is the investment threshold ratio for } I = 0, \quad \gamma_1 = -(t-s) + \frac{\sqrt{(t-s)^2 + 4su}}{2s} > 1, \text{ and}$$

$$a_1 = \frac{1}{\gamma_1 p_I^{\gamma_1} \delta_p}, \text{ using the constants } s \equiv \sigma_p^2 + 2\rho\sigma_p\sigma + \sigma^2, \quad t \equiv \delta - \delta_p, \text{ and } u \equiv \delta.$$

Using a deterministic electricity price as in Section 3.1 indicates that for  $P = 0.10$ , the investment cost threshold is  $C_I = 0.0809$  and  $A_2 = 1.1172 \times 10^{-5}$ , whereas the model with a stochastic electricity price reveals  $C_I = 0.0858$  for  $\sigma_p = 0.03$  and  $\rho = 0.80$ . For the latter, we obtain  $\gamma_1 = 7.0459$ ,  $a_1 = 1.4064$ , and  $p_I = 1.1654$ , and the corresponding option values to invest in the two cases are US\$0.85M and US\$0.62M, respectively. Intuitively, relatively low volatility in  $P$  along with high  $\rho$  reduces the scope for large cost savings from DG operation, thereby making it more attractive to invest sooner when both prices are stochastic. Relative to the case with a deterministic electricity price, when  $C$  decreases here,  $P$  also decreases, but slightly, due to the positive correlation between the two processes, which cuts the cost savings from on-site generation. Conversely, when  $C > P$ , the losses from on-site generation are reduced. However, since only savings from on-site generation are bounded, they are more affected; hence, this reduces the option value of waiting.

In contrast, when  $P$  is relatively volatile, there is greater chance of both exceptionally high and low cost savings from DG operation. In this case, it is beneficial for the microgrid to wait longer. Here, if  $C$  decreases, then  $P$  also decreases, but to a greater extent (reflecting its larger relative volatility), which results in losses due to on-site generation that are bounded as  $P$  cannot become negative. Alternatively, if  $C$  increases, then  $P$  also increases by a relatively greater amount, which results in savings from on-site generation that are unbounded as  $P$  can conceivably increase without limit. Consequently, this greater relative uncertainty increases the option value of the DG investment opportunity. For example, if  $\sigma_p = 0.12$ , then  $C_I = 0.0718$ , and the option value to invest is US\$1.32M (see Figs. 4-6). Finally, using the value for  $p_I$ , we

create an investment threshold boundary to indicate the highest possible natural gas generation cost that permits DG investment for each level of electricity price (see Fig. 7). As  $\sigma_p$  increases, *ceteris paribus*, the slope of the boundary increases, thereby reducing the size of the region in which investment is optimal.

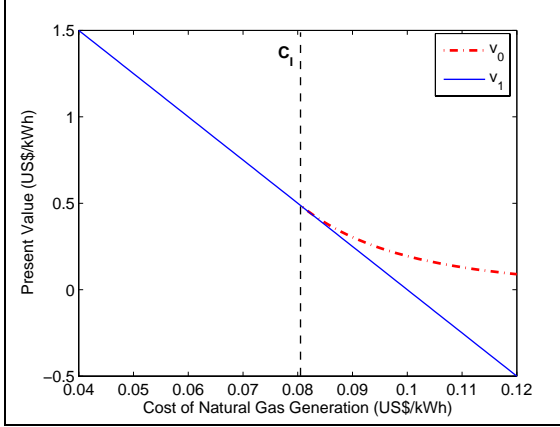


Figure 4. Value of Investment Opportunity (Deterministic Electricity Price and Zero Turnkey Cost).

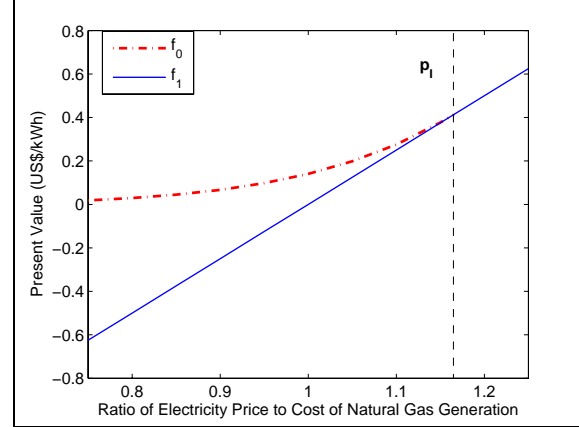


Figure 5. Value of Investment Opportunity (Low-Volatility Stochastic Electricity Price and Zero Turnkey Cost).

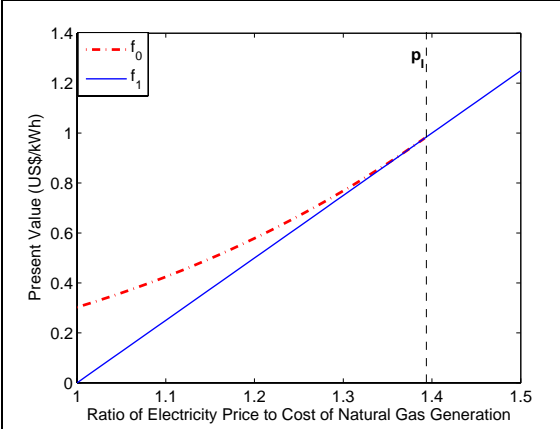


Figure 6. Value of Investment Opportunity (High-Volatility Stochastic Electricity Price and Zero Turnkey Cost).

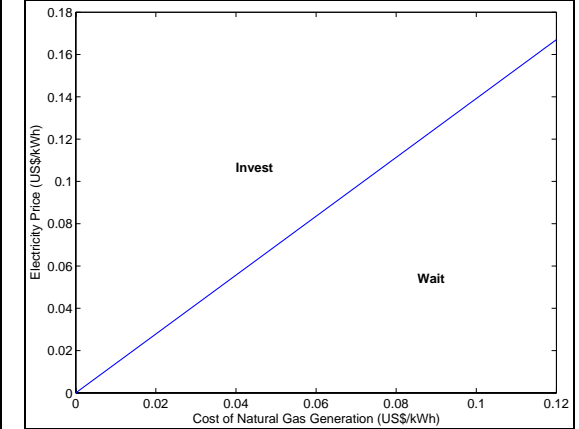


Figure 7. Investment Threshold Boundary for High-Volatility Stochastic Electricity Price.

#### 4. SUMMARY

The ongoing if fitful deregulation of electricity industries worldwide provides scope for decentralised decision-making as a means to improve economic efficiency. By giving incentives to market participants to act optimally, policymakers should aim to achieve outcomes in which energy resources are utilised consistent with their relative marginal values. In such a competitive environment, however, market participants must also account for price risk when making decisions, a dimension that was largely absent in the era of price regulated vertical integration. Therefore, the standard DCF approach for analysing investment and operations

must be either modified or replaced outright by techniques that explicitly factor in the roles of price volatility and managerial discretion.

This paper takes a real options approach to analyse the investment and operational decisions of a California microgrid. We find threshold fuel costs for triggering investment in DG, suspending and re-starting DG operations, and disconnecting from the electricity utility altogether. Greater operational flexibility makes DG investment more attractive for the microgrid, while the disconnection option is exercised only in the rare case of sustained natural gas cost decreases. An extension to the basic model to permit stochastic electricity prices (positively correlated with natural gas costs) indicates that relatively low (high) volatility in electricity prices increases (decreases) immediate investment in DG as the microgrid has less (more) chance of sustained cost saving increases from waiting longer. For future work, we intend to examine incremental investment under uncertainty in a portfolio of alternative DG technologies by a microgrid (Pindyck, 1989).

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<sup>1</sup> This is calculated by multiplying the natural gas fuel cost (US\$/kJ) by the heat rate (kJ/kWh) of the DG unit.

<sup>2</sup>  $\beta_2$  is simply the solution to the characteristic quadratic equation.

<sup>3</sup> The intuition behind these is similar to that for Equations (8) and (9).

<sup>4</sup> It should be noted that the shutdown and re-start costs are estimated using the annual variable operating and maintenance (O&M) cost for this DG unit as follows:  $\frac{US\$0.012}{kWh} \cdot 8760h \cdot 500kW = US\$52,560$ . In other

words, the approximate annual incremental O&M costs of this DG unit assuming constant output at rated capacity is US\$50,000. Since the optimal operating strategy of the microgrid is not known in advance, it is not clear how this additional amount should be allocated to each shutdown and re-start decision. As a convention, we assume that, on average, there will be ten such decisions during a year, which yields  $S = 5000$  and  $R = 5000$  as additional costs associated with operational flexibility.

<sup>5</sup> The DG investment opportunity is similar to a put option on natural gas generation, which increases in value with the volatility of the underlying asset since this makes extremely low prices more probable.